CANADIAN JOURNAL OF RESEARCH

VOLUME 27 NOVEMBER, 1949 NUMBER 6

— SECTION A —

PHYSICAL SCIENCES

Contents

	Page
Permeability Studies. IV. Surface Area Measurements of Zinc Oxide and Potassium Chloride Powders—*J. C. Arnell*	207
The Spectrum of Atomic Oxygen in a High Current A-C. Discharge—*W. Petrie, P. A. Monaghan, and P. A. Dolan*	213
Dielectric Properties of Some Animal Tissues at Meter and Centimeter Wave Lengths—*E. R. Laird and K. Ferguson*	218
A New High Light Gathering Power Spectrograph for Auroral Studies—*W. Petrie*	231

NATIONAL RESEARCH COUNCIL
OTTAWA, CANADA

CANADIAN JOURNAL OF RESEARCH

The *Canadian Journal of Research* is issued in six sections, as follows:

A. Physical Sciences	D. Zoological Sciences
B. Chemical Sciences	E. Medical Sciences
C. Botanical Sciences	F. Technology

For the present, Sections A, C, D, and E are to be issued six times annually, and Sections B and F, twelve times annually, each under separate cover. with separate pagination.

The *Canadian Journal of Research* is published by the National Research Council of Canada under authority of the Chairman of the Committee of the Privy Council on Scientific and Industrial Research. The *Canadian Journal of Research* is edited by a joint Editorial Board consisting of members of the National Research Council of Canada, the Royal Society of Canada, and the Chemical Institute of Canada.

Sections B and F of the *Canadian Journal of Research* have been chosen by the Chemical Institute of Canada as its medium of publication for scientific papers.

EDITORIAL BOARD

Representing
NATIONAL RESEARCH COUNCIL

DR. H. P. ARMES (*Chairman*),
Dean of the University,
University of Manitoba,
Winnipeg, Man.

* DR. G. H. HENDERSON,
Professor of Mathematical Physics,
Dalhousie University, Halifax.

DR. ROBERT NEWTON,
President, University of Alberta,
Edmonton, Alta.

DR. C. H. BEST,
The Banting and Best Department of Medical Research,
University of Toronto, Toronto.

Representing
ROYAL SOCIETY OF CANADA

DR. A. NORMAN SHAW,
Chairman, Department of Physics,
McGill University, Montreal.

DR. J. W. T. SPINKS,
Head, Department of Chemistry,
University of Saskatchewan,
Saskatoon.

} Section III

DR. H. S. JACKSON,
Head, Department of Botany
University of Toronto, Toronto.

DR. E. HORNE CRAIGIE,
Department of Zoology,
University of Toronto,
Toronto.

} Section V

Ex officio

DR. LÉO MARION, Editor-in-Chief,
Division of Chemistry,
National Research Laboratories,
Ottawa.

DR. H. H. SAUNDERSON,
Director, Division of Information Services,
National Research Council,
Ottawa.

Representing
THE CHEMICAL INSTITUTE OF CANADA

DR. H. G. THODE,
Department of Chemistry,
McMaster University,
Hamilton.

EDITORIAL COMMITTEE

Editor-in-Chief, DR. LÉO MARION
Editor, Section A, DR. A. NORMAN SHAW
Editor, Section B, DR. J. W. T. SPINKS / DR. H. G. THODE
Editor, Section C, DR. H. S. JACKSON
Editor, Section D, DR. E. HORNE CRAIGIE
Editor, Section E, DR. J. B. COLLIP
Editor, Section F, DR. J. A. ANDERSON / DR. A. NORMAN SHAW / DR. H. G. THODE

Manuscripts should be addressed:

Editor-in-Chief,
Canadian Journal of Research,
National Research Council, Ottawa, Canada.

* *Deceased.*

Canadian Journal of Research

Issued by THE NATIONAL RESEARCH COUNCIL OF CANADA

VOL. 27, SEC. A. NOVEMBER, 1949 NUMBER 6

PERMEABILITY STUDIES

IV. SURFACE AREA MEASUREMENTS OF ZINC OXIDE AND POTASSIUM CHLORIDE POWDERS[1]

BY J. C. ARNELL

Abstract

The use of the modified Kozeny equation for the measurement of the specific surfaces of fine powders has been extended to include a number of standard zinc oxides and a sample of potassium chloride. The specific surfaces of the zinc oxides had been measured by 10 other methods, and the data are included for comparison. Comparative low temperature adsorption data for the potassium chloride are also given. Satisfactory agreement with other methods was found.

Introduction

In several papers published within the past three years (1, 2, 3, 7), a method has been discussed for the determination of the specific surface of very fine powders by gas-permeability methods. This method depends on the use of a modified Kozeny equation, which contains an additional term to correct for 'slip'. This equation has been found to be of the greatest use in the form:

$$G = \frac{Ah\,\epsilon^2\,\rho_1}{kLS_v(1-\epsilon)}\left[\frac{\epsilon\,\rho_{H_0}g\overline{P}}{\eta S_v(1-\epsilon)} + \frac{8}{3}\sqrt{\frac{2R_0T}{\pi M}}\,\delta k_0\right], \qquad (1)$$

where G = rate of gas flow in grams per second,

A = cross-sectional area of the bed in square centimeters,

h = pressure head in centimeters of mercury,

ϵ = volume of pore space per unit volume (porosity),

$\rho_1 = \rho/P$ = density of the gas at unit pressure,

k = Kozeny's constant, which is approximately equal to 5,

L = length of the bed in centimeters,

S_v = specific surface of the powder in square centimeters per cubic centimeter,

[1] *Manuscript received July 8, 1949.*

Contribution from the Defence Research Chemical Laboratories, Ottawa, Canada. Also issued as D.R.C.L. Report No. 32.

ρ_{Hg} = density of mercury in grams per cubic centimeter,

g = acceleration due to gravity in centimeters per second per second,

$\overline{P} = (p_1 + p_2)/2$ = mean pressure in centimeters of mercury,

η = viscosity of the gas in poises,

R_0 = gas constant,

T = absolute temperature,

M = molecular weight of the gas,

δ = variable factor, having a value of approximately 0.9, and

k_0 = shape factor, whose value generally lies between 2 and 3 (5).

This equation has been tested against the data obtained from the flow of air through beds of finely ground quartz, inorganic pigment powders, and carbon blacks, and found to give values for the surface areas that showed good agreement with the values obtained by the use of other methods.

In this paper, the method has been applied to four samples of zinc oxide supplied by the New Jersey Zinc Company (of Pa.), Palmerton, Pennsylvania, U.S.A. About 15 years ago these samples were set aside for use in particle size investigations and their surface areas have been measured by many different methods in the intervening years. It was therefore of great interest to compare the results given by Equation (1) with the previous data.

Apparatus and Technique

The apparatus used for measuring the air flow through the different beds of zinc oxide has been described in detail in a previous paper in this series (2). The air flow through a bed of powder was measured at five different mean pressures, which were approximately 20, 30, 40, 50, and 65 cm. of mercury. The mass flows were then plotted against the corresponding mean pressures and the equation of the resultant straight line determined for each bed. These equations had the form:

$$G = B\overline{P} + C\,. \tag{2}$$

The specific surface was then calculated in two different ways. In the first method, the slope of the line, B, was equated to the first term on the right-hand side of Equation (1), which is equivalent to the original Kozeny equation, and in the second method, the intercept, C, was equated to the pressure independent term of the modified equation.

Experimental Results

The specific surfaces were calculated by the two methods from the data obtained on four beds of each sample of zinc oxide. The experimental results

are listed in Table I, together with the porosities of the beds calculated using a zinc oxide density of 5.67 (4). The areas in square meters per gram have been calculated by dividing the specific surface by the density and are also included for comparison.

TABLE I

SURFACE AREAS OF ZINC OXIDE POWDERS

Sample	Porosity	Specific surface, $m^2/cc.$	
		Slope (B)	Intercept (C)
Kadox Black Label-15	0.698	38.9	47.9
F1601	0.660	43.0	45.2
	0.706	50.9	43.6
	0.695	46.4	40.8
		44.8	44.4
		7.9 $m^2/gm.$	7.8 $m^2/gm.$
XX Red-72	0.647	36.0	37.0
K1602	0 630	33.8	33 9
	0.578	29.7	34.9
	0.617	38.1	39.1
		34.4	36.2
		6.1 $m^2/gm.$	6.4 $m^2/gm.$
XX Red-78	0.569	16.4	15.5
G1603	0.557	15.0	14.3
	0.570	16.3	16.2
	0.536	16.6	16.6
		16.1	15.7
		2.8 $m^2/gm.$	2.8 $m^2/gm.$
Reheated Superfine	0.458	3.6	4.0
KH 1604	0.432	4.3	5.0
	0.424	3.7	4.2
	0.468	3.7	3.8
		3.9	4.3
		0.6_9 $m^2/gm.$	0.7_6 $m^2/gm.$

In addition to the above work, the specific surface of a sample of potassium chloride was measured by this method. The details of the preparation of the sample and the low temperature adsorption data have been published elsewhere (10). In brief, the sample was prepared by atomizing a solution of potassium chloride and collecting the resulting aerosol electrostatically. The experimental results are listed in Table II, together with the BET surface areas determined from low temperature adsorption isotherms (8).

TABLE II

SURFACE AREA OF FINELY DIVIDED POTASSIUM CHLORIDE

Permeability method	Specific surface, $m^2/cc.$	
Porosity	Slope (B)	Intercept (C)
0.481	4.1_2	4.0_8
0.473	4.0_1	4.0_5
0.471	3.5_4	3.5_2
	3.8_9	3.8_8
	1.9_6 $m^2/gm.$	1.9_6 $m^2/gm.$

Low temperature adsorption method (BET) (after Keenan)

Gas	Surface area, $m^2/gm.$			
	83° K.	84° K.	85° K.	90° K.
Nitrogen	—	2.19	2.21	2.21
Argon	2.83	—	2.92	2.79
Oxygen	—	—	2.76	2.63

Discussion

Through the kindness of Dr. C. E. Barnett of the New Jersey Zinc Company, the following table (Table III) summarizing all the surface area data obtained with the zinc oxide samples was made available for inclusion here.

Some of these data have been published previously (6, 8, 9), but much of it is new. The sedimentation results were measured by A. E. Jacobsen of the National Lead Company, Titanium Pigments Division. The electron microscope, turbidimetric, and infrared transmission measurements were made by the New Jersey Zinc Company, while the air permeability data using the single Kozeny equation were obtained by both the New Jersey Zinc Company and the National Bureau of Standards, Washington.

In general the results from Equation (1) are in fair agreement with those obtained from liquid permeability measurements; this indicates that the modified Kozeny equation adequately compensates for the 'slip' that occurs with gas flow through beds of very fine powders. These results also agree reasonably well with the gas adsorption data. Some of the other methods have been known in general to give low values for the surface area, and this is borne out here. With the larger particles, as in KH 1604, much better agreement is obtained between methods, owing probably to such factors as the virtual disappearance of the 'slip' factor.

The data on the potassium chloride sample are included as another example of the satisfactory agreement between the surface area measurements obtained by low temperature gas adsorption and air permeability corrected for slip.

TABLE III

SURFACE AREAS OF ZINC OXIDE PIGMENTS USING DIFFERENT METHODS

	Light microscope	Electron microscope	Liquid adsorption	Sedimentation	Permeability			Gas adsorption	Dark field count	Turbidimeter	Infrared transmission
					Liquid	Air					
						Kozeny equation	Equation (1) Table II				
Kadox Black Label-15 F-1601											
d_1	0.15	0.10	—	—	—	—	—	—	—	0.18	—
d_2	0.28	0.18	0.20	0.30	0.12	0.26	0.14	0.11	—	—	—
S, m²/gm.	3.8	6.0	5.4	3.5	8.9	4.1	7.8	9.5	—	—	—
D	0.21	0.14	—	—	—	—	—	—	0.14	—	—
$N \times 10^{10}$	2042	—	—	—	—	—	—	—	7257	—	—
d_4	0.37	—	—	—	—	—	—	—	—	—	0.27 / 0.34
XX Red-72 K-1602											
d_1	0.19	0.12	—	—	—	—	—	—	—	0.23	—
d_2	0.34	0.23	0.25	0.37	0.15	0.32	0.17	0.12	—	—	—
S, m²/gm.	3.1	4.6	4.3	2.9	7.2	3.3	6.2	8.8	—	—	—
D	0.25	0.15	—	—	—	—	—	—	0.16	—	—
$N \times 10^{10}$	1139	—	—	—	—	—	—	—	4200	—	—
d_4	0.42	—	—	—	—	—	—	—	—	—	0.44
XX Red-78 G-1603											
d_1	0.30	0.25	—	—	—	—	—	—	—	0.36	—
d_2	0.79	0.48	0.56	0.76	0.25	0.62 / 0.62	0.38	0.27	—	—	—
S, m²/gm.	1.3	2.2	1.9	1.4	4.3	NBS-1.7 NJZ-1.7	2.8	3.9	—	—	—
D	0.49	0.35	—	—	—	—	—	—	0.26	—	—
$N \times 10^{10}$	153	—	—	—	—	—	—	—	1076	—	—
d_4	1.1	—	—	—	—	—	—	—	—	—	0.54
Reheated Superfine KH-1604											
d_1	1.0	—	—	—	—	—	—	—	—	—	—
d_2	1.9	—	5.3	2.1	1.25	1.63	1.5	1.61	—	—	—
S, m²/gm.	0.6	—	0.2	0.5	0.85	0.65	0.71	0.66	—	—	—
D	1.4	—	—	—	—	—	—	—	0.82	—	—
$N \times 10^{10}$	6.6	—	—	—	—	—	—	—	32	—	—
d_4	2.2	—	—	—	—	—	—	—	—	—	—

NOTE: d_1 = *average diameter* ($\Sigma nd/\Sigma n$) *in microns,*

d_2 = *mean volume-surface diameter* ($\Sigma nd^3/\Sigma nd^2$) *in microns,*

S = *surface area in square meters per gram,*

D = *mean volume diameter* ($\sqrt[3]{\Sigma nd^3/\Sigma n}$) *in microns,*

N = *number of particles per gram and given by* $N = 1/\mu D^3$, *where* μ = *specific gravity,*

d_4 = *weight mean diameter* ($\Sigma nd^4/\Sigma nd^3$) *in microns.*

NBS – National Bureau of Standards.

NJZ – New Jersey Zinc.

Acknowledgments

The author would like to express his thanks to Dr. C. E. Barnett and the New Jersey Zinc Company (of Pa.), Palmerton, Pennsylvania, for supplying the zinc oxide samples and their interest and help in making available the existing surface area data, and to Dr. A. G. Keenan of the National Research Council of Canada for supplying the potassium chloride sample.

References

1. ARNELL, J. C. Can. J. Research, A, 24 : 103. 1946.
2. ARNELL, J. C. Can. J. Research, A, 25 : 191. 1947.
3. ARNELL, J. C. and HENNEBERRY, G. O. Can. J. Research, A, 26 : 29. 1948.
4. BARNETT, C. E. Private communication.
5. CARMAN, P. C. Trans. Inst. Chem. Engrs. (London), 15 : 150. 1937.
6. CARMAN, P. C. Symposium on new methods for particle size determination in the subsieve range, p. 24. Am. Soc. Testing Materials. 1941.
7. CARMAN, P. C. and ARNELL, J. C. Can. J. Research, A, 26 : 128. 1948.
8. EMMETT, P. H. Symposium on new methods for particle size determination in the subsieve range, p. 95. Am. Soc. Testing Materials. 1941.
9. EWING, W. W. J. Am. Chem. Soc. 61 : 1317. 1939.
10. KEENAN, A. G. In press.

THE SPECTRUM OF ATOMIC OXYGEN IN A HIGH CURRENT A-C. DISCHARGE[1]

By W. Petrie, P. A. Monaghan,[2] P. A. Dolan[3]

Abstract

The atomic oxygen spectrum has been produced in an a-c. discharge of current densities between 50 m.a. per cm^2 and 1.3 amp. per cm^2, and lines arising from the lower permitted levels of the atom are present in this spectrum. In view of the fact that the discharge is excited chiefly by electron collisions, and also that the aurora appears to be the result of collisional processes, the auroral spectrum should be examined carefully to determine whether or not these same permitted atomic oxygen lines are present.

Introduction

There is some doubt regarding the appearance of atomic oxygen lines in the auroral spectrum. A number of lines that originate from the lower levels of the atom fall in the near infrared, and this region of the auroral spectrum has not been too well resolved. Since N_2^+ bands are features of the spectrum, evidently sufficient energy is available to populate the lower levels of the oxygen atom. There is much evidence that the auroral radiations are the result of collisional processes, and it seemed worthwhile to examine the spectrum of oxygen in an a-c. discharge in which the mechanism of excitation is chiefly electron collision. If lines from the lower levels of the oxygen atom are present, we might expect these same lines to appear in the auroral spectrum. Discharge spectra of oxygen have been studied by a number of workers including Frerichs (1), Hopfield (2), and Kvifte and Vegard (3). These spectra, however, only partially answer our question. Several of the lines in which we are interested are listed in the measures of Frerichs (1) and Kvifte and Vegard (3), but the former spectra are chiefly of the ionized oxygen atom, and those studied by Kvifte and Vegard (3) do not cover the spectral region in which a number of the lower level lines fall.

Description of the Apparatus

The discharge tube was 69 cm. long with an inside diameter of 1.4 cm., and the tube was surrounded with a water jacket of diameter 2.5 cm. The ends of the tube were flared out to 6 cm. diameter, and disk-shaped aluminum electrodes 12 cm. in diameter and 0.9 cm. thick were waxed to each end. A 1.4 cm. diameter hole was drilled through one electrode and a quartz window waxed to the outside, thus sealing the discharge chamber. In order to cool the ends of the tube and the electrodes during operation of the discharge, both electrodes were milled in radially to a distance of 2.5 cm. and made

[1] *Manuscript received May 7, 1949.*
Contribution from the Department of Physics, University of Saskatchewan, Saskatoon, Sask.

[2] *Now with the Dept. of Mines and Resources.*
[3] *Now attending the University of British Columbia.*

waterproof by means of a rubber ring attached to the periphery. Inlet and outlet water cocks were attached to each electrode. The discharge chamber was evacuated by means of a rotary oil pump, and the discharge operated from a power supply that is rated at 2 kva. Two prism spectrographs were used to photograph the spectra, one a medium quartz Hilger instrument, and the other a spectrograph constructed in the Physics Shop at the University of Saskatchewan.

One liter of high purity oxygen was purchased from the Matheson Company of New Jersey, and a supply was also obtained by heating a flask containing potassium permanganate.

Discussion of the Spectra

A number of spectra were photographed in the region 3500 to 9000 angstroms, after the discharge tube had been flushed several times with the pure oxygen. A variety of voltages and currents were used, but the majority of the spectra were photographed when the discharge current density was around 50 m.a. per cm^2. The pressure was not measured, but was estimated to be several millimeters of mercury.

It is not our purpose to give a complete discussion of the spectra; in fact, the dispersion is too low for a careful analysis. However, we wish to point out that lines from the lower levels of the oxygen atom are certainly present in these spectra, as well as a few weak lines from singly ionized atoms.

It is worthwhile to compute the theoretical intensities of the low level lines if these are to be searched for in the auroral spectrum. The intensity of a spectral line from a source in thermodynamic equilibrium may be expressed in several ways. In terms of the Einstein transition coefficient the intensity of a spectral line is $I = N A h \nu$, where N is the number of atoms in the upper level from which the line arises, and A is the Einstein transition coefficient appropriate to the level involved. The intensity of a spectral line may also be expressed in the form $I = \frac{64\pi^4\nu^4}{3c^3}\frac{N}{\omega}[(a|P|b)]^2$, where ω is the statistical weight of the level from which the transition arises. This expression is similar to the one that gives the radiation from the classical oscillator, except for the factors ω and $[(a|P|b)]^2$; the latter is the square of the average electric moment for the levels involved. This factor is an integral involving wave functions and may be reduced to a product $\frac{e^2a^2}{4l^2-1}s\rho^2$, where a is the radius of the first Bohr orbit, l is the larger of the initial or final values of the orbital quantum number of the "jumping electron" associated with the two electron configurations involved in the transition, s is the theoretical line strength, and ρ is the radial quantum integral, i.e., $\rho = \int R_{nl}\, R_{n'l'}\, r^3\, d\, r$, where R_{nl} and $R_{n'l'}$ are the radial wave functions associated with the two electron configurations. Since we are dealing with the radiation from a source in thermodynamic equilibrium, the number of atoms in any level in terms of the

number in the ground level is expressed by the Boltzmann relation, i.e., $N = N_0 \frac{\omega}{\omega_0} e^{-\chi/kT}$, where N_0 is the number of atoms in the ground level, ω_0 is the statistical weight of the ground level, and χ the excitation potential of the level from which the transition arises. In these expressions, ν, c, e, h, k, and T have their usual meanings.

Then

$$I = NAh\nu = \frac{64\pi^4\nu^4}{3c^3} \frac{N_0}{\omega_0} e^{-\chi/kT} \frac{e^2 a^2}{4l^2 - 1} s\rho^2 . \quad (1)$$

The calculation of ρ requires a knowledge of the radial wave functions. Slater (6) has shown how to obtain approximate hydrogenlike radial wave functions, and Menzel and Aller (4) have shown how to use these functions in the calculation of ρ. Using the Slater method, the integral for ρ becomes

$$\rho^2 = \left[\frac{2(Z - \sigma_1)}{n_1}\right]^{2n_1+1} \left[\frac{2(Z - \sigma_2)}{n_2}\right]^{2n_2+1} \frac{1}{\Gamma(2n_1)} \frac{1}{\Gamma(2n_2)} \left[\frac{\Gamma(n_1 + n_2 + 2)}{\left\{\left(\frac{Z - \sigma_1}{n_1}\right) + \left(\frac{Z - \sigma_2}{n_2}\right)\right\}^{n_1 + n_2 + 2}}\right]^2 . \quad (2)$$

In this formula Z is the atomic number, σ_1 and σ_2 are the screening constants for the two electron configurations involved in the transition, and n_1 and n_2 are the effective quantum numbers of the "jumping electron" in each electron configuration. The quantities σ and n are formed from the information given in Slater's paper. We have computed ρ values from Equation (2), and the results are given in Table I.

TABLE I

THEORETICAL OXYGEN LINE INTENSITIES

Transition	J'	J	λ	χ_n	s	l	ρ	I	M.I.T.
$2p^33s(^5S^0) - 2p^33p(^5P)$	2	3	7772	10.69	7.1	1	6	1.0	1.0
$2p^33s(^3S^0) - 2p^33p(^3P)$	1	2	8446	10.94	5.0	1	6	0.24	2.0
$2p^33p(^3P) - 2p^34s(^3S^0)$	3	2	11300	11.79	7.1	1	4.7	0.006	—
$2p^33p(^3P) - 2p^34s(^5S^0)$	2	1	13163	11.88	5.0	1	4.7	0.002	—
$2p^33p(^3P) - 2p^33d(^5D^0)$	3	4	9266	12.03	54.0	2	5.8	0.015	0.03
$2p^33p(^3P) - 2p^33d(^3D^0)$	2	3	7987	12.04	4.2	2	5.8	0.002	0.05
$2p^33s(^5S^0) - 2p^34p(^5P)$	2	3	3947	12.23	7.1	1	4.7	0.10	0.3
$2p^33s(^3S^0) - 2p^34p(^3P)$	1	2	4368	12.30	5.0	1	4.7	0.04	1.0
$2p^33p(^3P) - 2p^33s(^3D^0)$	2	3	7995	12.54	4.2	1	6	0.003	0.05
$2p^33p(^5P) - 2p^35s(^5S^0)$	3	2	6456	12.61	7.1	1	2.9	0.002	0.5
$2p^33p(^3P) - 2p^35s(^3S^0)$	2	1	7254	12.64	5.0	1	2.9	0.0008	0.015
$2p^33p(^3P) - 2p^36s(^5S^0)$	3	2	5437	12.96	7.1	1	2.4	0.0009	0.2

The method of obtaining the theoretical line strength s has been discussed by Petrie (5). Using Equation (1) the intensities of the lines arising from the lower permitted levels of the oxygen atom have been computed and are

listed in Table I, along with other relevant data. Only the strongest line in each multiplet is given. The intensity of the line λ 7772 which arises from the lowest excited permitted level is arbitrarily made unity, and the other intensities are in terms of the intensity of this line. This manner of expressing the intensities eliminates the necessity of including the constant factor $\frac{64\pi^4}{3c^3}\frac{N_0}{\omega_0}e^2a^2$. An explanation of Table I follows.

Column 1. The electron configurations and terms from which the line arises. The configuration of lower energy is written first.

2. The J values of the two levels from which the line arises.
3. The wave length of the line, in angstroms.
4. The excitation potential in electron volts of the upper level involved in the production of the line.
5. The theoretical line strength s.
6. The l value of the "jumping electron" associated with each transition.
7. The ρ values for each transition.
8. The calculated line intensity.
9. Intensities listed in the M.I.T. wave length tables. The intensities in Columns 7 and 8 are in terms of the intensity of the line λ 7772.

Concluding Remarks

All the lines listed in Table I in the region 3500 to 9000 angstroms appear in our spectra. Hence, a careful search should be made of the auroral spectrum to determine whether or not these same lines are present. Vegard (7) has listed the line λ 4368 as being present in the auroral spectrum. Unless there is something very strange about the excitation mechanism that operates in the upper atmosphere, other lines listed in Table I should be features of this spectrum. It is hoped that a new auroral spectrograph constructed at the University of Saskatchewan will settle the question of the appearance or non-appearance of these oxygen lines. It is not expected that the intensities of the oxygen lines in discharge spectra or in the auroral spectrum will agree exactly with the theoretical intensities that have been calculated. In discharge tubes and in the upper atmosphere there are departures from thermal equilibrium. Nevertheless, the theoretical intensities will be a useful guide when searching for these lines in the auroral spectrum.

Acknowledgments

The authors wish to thank Prof. E. L. Harrington for his generous assistance during the construction of the discharge tube. This investigation was supported by a grant from the National Research Council of Canada.

References

1. FRERICHS, R. Phys. Rev. 34 : 1239. 1929.
2. HOPFIELD, J. J. Phys. Rev. 37 : 160. 1931.
3. KVIFTE, G. and VEGARD, L. Geofys. Publikasjoner. Norske Videnskaps-Akad. Oslo, 17 : No. 1. 1947.
4. MENZEL, D. H. and ALLER, L. H. Astrophys. J. 94 : 436. 1941.
5. PETRIE, W. Can. J. Research, A, 25 : 293. 1947.
6. SLATER, J. C. Phys. Rev. 36 : 57. 1930.
7. VEGARD, L. Geofys. Publikasjoner. Norske Videnskaps-Akad. Oslo, 13 : No. 5. 1941.

DIELECTRIC PROPERTIES OF SOME ANIMAL TISSUES AT METER AND CENTIMETER WAVE LENGTHS[1]

By E. R. Laird and K. Ferguson

Abstract

Measurements have been made of the dielectric constant and the absorption coefficient of some animal tissues at λ 1.72 m. by the Drude–Cooledge method, and at λ 9.5 cm. and λ 3.2 cm. by the reflection with change of phase method as developed by von Hippel. The dielectric constant of beef fat changed from about 9.5 at λ 1.72 m. to 3.5 at λ 3.2 cm., that of bone remained about constant between 6.5 and 7, that of beef bone marrow was reduced from 20 to 3.7, and that of frog muscle from 135 to 35. The absorption coefficient increased from 0.026 to 0.79 per cm. for fat, from 0.0028 to 0.56 for bone, from 0.028 to 0.9 for bone marrow, and from 0.08 to between 5 and 6 for frog muscle in the same range. Uncertainties of measurement and variation between samples are in general large. The dielectric constant found for muscle at λ 1.72 cm. is higher than a value given by Osswald at λ 3 m.

Introduction

The work reported here was undertaken as part of a project, proposed by Dr. R. C. Dearle, to examine the biological effects of electromagnetic radiation of wave length less than 2 m. for possible clinical application. Relative heating effects in the special case of the use of condenser fields had already been studied, and McLennan and Burton (9, 10) had also examined this case theoretically, and given the relation connecting conductivity, dielectric constant, and frequency for optimum effect. Schereschewsky (13) made some measurements on dielectric constant at about 2 m. and Osswald (11) on both dielectric constant and conductivity of a number of animal tissues at 3 m. At shorter wave lengths the direct use of radiation becomes possible, and for this case also data on the dielectric constant and either the conductivity or absorption coefficient are desirable. Such measurements have now been made at wave lengths of approximately 1.72 m., 9.5 cm., and 3.2 cm. on muscle, fat, bone, and bone marrow. Some observations have also been made on liver and on water, and especially at λ 1.7 m. any appearance of nonheating effects was noted.

Method and Equipment

I. λ 1.72 m.

A consideration of the material to be used, and simplicity of construction, led to the choice of the Drude-Cooledge (5) method modified to suit present equipment. This method has been used in more recent years by Slätis (14) and found satisfactory for materials having an absorption index as high as 0.25. Its use has been criticized by Abadie (1) as unsatisfactory for highly absorbing substances, and a study showed that at times corrections should be made to the previously used formulae.

[1] *Manuscript received June 28, 1949.*

Contribution from the Department of Physics, The University of Western Ontario, London, Ont. This work was supported in part by the Ontario Cancer Treatment and Research Foundation.

The method uses a Lecher wire system, as in Fig. 1. B is a fixed bridge to which a copper–constantan thermocouple is attached. B' is a sliding bridge, and at A is a glass container with sealed-in platinum electrodes for holding

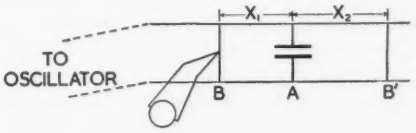

FIG. 1.

the materials to be used. The method of direct coupling to a lossy line from the continuous wave oscillator was tried, and seemed satisfactory. The bridge at B' is adjusted for a maximum response of the galvanometer, x_1, x_2, and the galvanometer deflection G_2 are measured, B' is again adjusted with the container removed, and the galvanometer deflection read, G_1. The logarithmic decrement of the circuit without the container, δ, and the constants of the container-condenser, may be determined separately. From these data the dielectric constant and absorption coefficient of the material may be found.

Theory shows, see appendix, that when the conductivity of the material being tested is low, resonance occurs when

$$\cot \theta_1 + \cot \theta_2 = \omega CZ = \omega Z(C' + \epsilon C_0), \tag{1}$$

where $\theta_1 = \frac{2\pi x_1}{\lambda}$, $\theta_2 = \frac{2\pi x_2}{\lambda}$, Z is the characteristic impedance of the line, and C is the capacity across it at A. Following Drude, one may put C equal to $C' + \epsilon C_0$, where C' is the capacity due to the wires external to the container, C_0 is the internal capacity with air as medium, and ϵ is the dielectric constant of the material being used.

The theory shows also that for low conductivity the index of absorption is given by

$$\frac{4\kappa}{1 - \kappa^2} = \frac{C' + \epsilon C_0}{\epsilon C_0} \cdot \frac{\delta}{\cot \theta_1 + \cot \theta_2} \left[(1 + \cot^2 \theta_1) \sqrt{\frac{G_1}{G_2}} - \frac{2}{\lambda} \left(\frac{x_1}{\sin^2 \theta_1} + \frac{x_2}{\sin^2 \theta_2} \right) \right] \tag{2}$$

The absorption coefficient 2α is $\frac{4\pi n \kappa}{\lambda}$.

The oscillator was an R.C.A.F. A.S.V. transmitter modified for continuous wave transmission. It was used at a frequency close to 175 Mc. per sec., and with constant voltage on the tube the output remained desirably constant.

The Lecher wires were of tinned copper 1.23 mm. diameter, centers 1.14 cm. apart. The sliding bridge B' was a silver disk 2.4 cm. diameter, with slots fitting closely over the Lecher wires, and could be held in position by the backing of a heavier brass bridge. The positions of the bridges and the

container were read either on a meter stick that could be laid for the purpose on supports close to the wires, or by means of a cathetometer fixed to be used horizontally at a distance of about 3 m. A correction was applied to the observed position of the thermocouple bridge B. Its effective position was taken to be at the distance $\lambda/2$ from the resonance position of the bridge B' when no container was present. $\lambda/2$ was taken as the distance between the first and second resonance settings of B', and could also be checked by a frequency meter. The correction to the position of B varied between 0.1 and 0.5 cm. for different thermocouples. It will be noted that this correction assumes that the impedance of B' is negligible.

The galvanometer zero was taken as that reading obtained when a short was put across the Lecher wires at a distance $\lambda/4$ from the bridge B. In this case the impedance looking into the circuit tends to infinity, and the current through the thermocouple bridge is a minimum. The use of this reading as a galvanometer zero assumes that either there is no constant phase relation between this bridge current and the additional current flowing when the circuit is at resonance, or that they are about $\pi/2$ apart in phase. The high frequency resistance of the bridge was computed to be about 0.13 ohm, and an estimate of the inductive reactance was 8 ohms. The additional current flowing at resonance will be in phase with the voltage, so the assumption mentioned appears justified. Values of this order for the bridge impedance also account for the observed difference between the bridge position and what has been called its effective position.

Equation (2) assumes that there is the same voltage at B when each of the two maximum deflections is observed. An estimate of the change in admittance at B gave in the extreme case 10%. The oscillator was connected to the Lecher wire system through lossy cable as stated above. Some tests were made on the effect of altering the point of connection, and it was found to be negligible. However, in the case of muscle it was considered that G_1/G_2 might be small by 10%.

There was the further experimental source of error in obtaining G_2 owing to changes in room temperature. The thermocouple bridge was protected by a small mica jacket, and the second junction was in the shielded cable leading to the galvanometer, but the heavy Lecher wires were unprotected, and at times this caused great difficulty.

Equation (1) shows that for a constant value of ωZ there should be a linear relation between ϵ and the sum of the cotangents, $\cot\theta_1 + \cot\theta_2$. This was initially tested by measuring x_1 and x_2 at resonance with the materials air, benzene, ethyl ether, 50% acetone, and water in the container, and was found to hold. The constants of the container finally used were found from observations in which air, benzene, ethyl ether, and water were the standards, and the results were $C_0 = 0.0315$ and $C' = 0.075$ e.s.u. This container had an internal diameter of 5 mm. The ends of the platinum leads were hammered out to make plates about 3 mm. in diameter, and the tube wall was slightly

expanded at this point so that they were separated by 5 mm. Fig. 2 shows the general form. The logarithmic decrement was determined at intervals from a resonance curve, and was found to remain substantially constant at

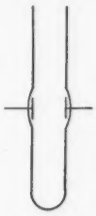

FIG. 2.

0.0070 ± 0.0001. The breadth of the resonance curve at the second maximum was, within experimental error, double that at the first maximum; this indicated that the damping due to the bridges was relatively negligible. The computed value for a circuit of the given dimensions of copper is 0.0044, for one of tin 0.0112.

These constants, δ, C', and C_0, along with the observations on the test material, suffice to determine ϵ and κ in many cases. For materials of higher conductivity Equations (1) and (2) require correction terms that depend on R, the resistance of the test material in the container-condenser (see Equations (7), (8), and (10) of the Appendix). The value of R was estimated from the value of κ given by (2). This estimate was then used in the complete formulae. As the maximum correction was not more than 5%, a second approximation was not deemed necessary.

II. λ *9.5 cm. and* λ *3.2 cm.*

The method used at these two wave lengths was essentially that described by Roberts and von Hippel (12) and is indicated by the block diagram—Fig. 3.

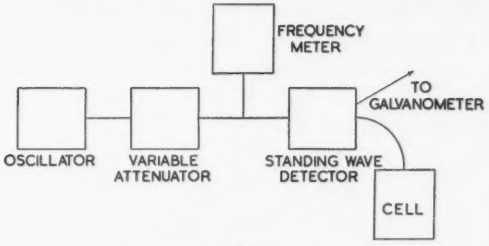

FIG. 3.

At λ 9.5 cm. the source was a Sutton tube, and was connected to a coaxial system. The cell had internal diameter 7 mm., the axis 2.94 mm., and was rather inconveniently small. At λ 3.2 cm. the source was a Shepard tube, and it fed into service size wave guide. The right angle bend was provided by a silver plated flexible piece of wave guide. In both cases the experiment consists in determining the depth d of the test material, the position of

the minimum on the slotted line, the ratio E_{min}/E_{max} of the field, and the frequency. Theory (L.C.) shows that

$$\frac{\tanh \gamma_2 d}{\gamma_2 d} = -j \frac{\lambda}{2\pi d} \frac{\frac{E_{min}}{E_{max}} - j \tan \frac{2\pi x_0}{\lambda}}{1 - j \frac{E_{min}}{E_{max}} \tan \frac{2\pi x_0}{\lambda}}, \tag{3}$$

where $\gamma_2 = \alpha_2 + j\beta_2$ is the propagation factor in the test medium, x_0 is the distance of the first minimum from the boundary surface, and λ is the wave length in air in the coaxial, or in the wave guide. The real part of the dielectric constant,

$$\epsilon' = \frac{\lambda_0^2}{\lambda_c^2} + \frac{\lambda_0^2}{4\pi^2} (\beta^2 - \alpha^2), \tag{4}$$

where λ_c is the cutoff wave length for the wave guide, and is infinity for the coaxial, and λ_0 is the wave length in free space corresponding to the given frequency. The absorption coefficient for the coaxial is $2\alpha_2$; for the wave guide it is also approximately $2\alpha_2$, but the more accurate value is

$$2\alpha_2 \frac{\beta_2 \lambda_0}{2\pi n},$$

where

$$n^2 = \tfrac{1}{2}(\epsilon' + \sqrt{\epsilon'^2 + \epsilon''^2}) \quad \text{and} \quad \epsilon'' = \alpha_2 \beta_2 \frac{\lambda_0^2}{2\pi^2}. \tag{5}$$

Equation (3) is not directly solvable. If it is written in the form $\frac{\tanh \gamma_2 d}{\gamma_2 d} = X + jY$, where X and Y are obtained from the data, then when $\alpha_2 d$ is large, so that $\tanh \alpha d \doteqdot 1$, the left hand side becomes $\frac{1}{\alpha_2 d + j\beta_2 d}$, and

$$\alpha_2 = \frac{X}{(X^2 + Y^2)d}, \qquad \beta_2 = \frac{-Y}{(X^2 + Y^2)d}.$$

The value for β_2 obtained is fairly accurate, but since X is small in this case and could not be as accurately determined, this was not so good a procedure for obtaining the absorption coefficient. When $\alpha_2 d$ is small, so that $\tanh \alpha_2 d \doteqdot \alpha_2 d$, and $\alpha_2 d^2 \tan \beta_2 d \ll \beta_2 d$, Equation (3) approximates to

$$\frac{\tan \beta_2 d}{\beta_2 d} = X + \frac{Y}{X}\left(Y + \frac{\alpha_2}{\beta_2}\right)$$

$$\alpha_2 d = \frac{Y \beta_2 d}{X(1 - \beta_2 d \tan \beta_2 d) - 1}.$$

Tables giving $\frac{\tan x}{x}$ for values of x are available in N.D.R.C. Contract OEMsr-191, Report IX, and are very convenient. If necessary an assumed value for $\frac{\alpha_2}{\beta_2}$ may be used in getting $\frac{\tan \beta_2 d}{\beta_2 d}$, and a further approximation made.

For intermediate cases, and in general in the checking of the accuracy of the solution, the forms

$$X = \frac{\alpha_2 d \sinh 2\alpha_2 d + \beta_2 d \sin 2\beta_2 d}{(\alpha_2^2 d^2 + \beta_2^2 d^2)(\cosh 2\alpha_2 d + \cos 2\beta_2 d)}$$

$$Y = \frac{\alpha_2 d \sin 2\beta_2 d - \beta_2 d \sinh 2\alpha_2 d}{(\alpha_2^2 d^2 + \beta_2^2 d^2)(\cosh 2\alpha_2 d + \cos 2\beta_2 d)}$$

given by Cripwell and Sutherland (6) were found convenient.

The standing wave ratio E_{min}/E_{max} could be determined to within 1%. The greatest experimental error lay in the determination of d for substances that were neither solid nor fluid and that could not be shaped accurately. The method used was to measure with a depth gauge the total depth of the terminal cup, and the distance to the top of the material. In some cases a piece of mica 0.02 mm. thick was placed on top of the sample material to provide a more definite base for measurement, but if surface irregularities are present, the mica position gives a maximum value of the depth. The value of d is used not only directly in Equation (3), but indirectly in getting x_0. This quantity is found from observations on the position of the minimum with the cup empty, X', and with the test material in place, X''. Then $x_0 = X'' - d - X'$. In the experiments at λ 9.5 cm., individual readings were made to 0.05 mm. In those at λ 3.2 cm. the X'' and X' could be measured to 0.01 or 0.02 mm., but the uncertainty in the value of d for the materials used was of the order of 0.1 mm. or more, and this introduced a large proportional error in x_0 in the observations with muscle. The first minimum observable in the λ 3.2 cm. set-up was six wave lengths from the base of the cup, and any slight change in frequency made an observable change in the readings X'' and X'. The frequency meter was read immediately before and after the other observations; under good conditions the change during a run was not more than 0.2 Mc. per sec.

In order to make observations on water at λ 9.5 cm. a portion of the cup was closed by a piece of polystyrene. This procedure might have improved the accuracy of the determination of d at λ 3.2 cm.

Results and Discussion

The results for muscle, fat, bone, bone marrow, and water at or near room temperature are shown in Table I. The power absorption coefficient is, as stated earlier, the same as the quantity commonly designated 2α, and is given in double nepers per centimeter. In all cases material designated as beef was from refrigerated supply; frog muscle, rabbit fat, and cat bone marrow were from freshly killed animals. A large number of samples of muscle were used. The spread in values obtained includes errors in measurement and real differences between the samples greater than the experimental error. Especially at λ 3.2 cm., these differences seemed to follow differences in the amount of fluid in the sample. In a general way the two constants were slightly higher for beef muscle than for frog, but the difference was not greater

TABLE I

Material	Dielectric constant			Absorption coefficient		
	1.72 m.	9.5 cm.	3.2 cm.	1.72 m.	9.5 cm.	3.2 cm.
Muscle						
Beef	147 ± 11	54 ± 6	37.5 ± 4	0.087 ± 0.011	1.54 ± 0.14	5 – 7
Frog	135 ± 4		34.5 ± 4	0.079 ± 0.005		5 – 6
Fat						
Beef	9.5 ± 2.2	3.44 ± 0.05	3.5 ± 0.3	0.026 ± 0.005	0.15 ± 0.02	0.79 ± 0.06
Rabbit	23 ± 1			0.036		
Bone	6.6 ± 1		6.86	0.0028		0.56
Bone marrow						
Beef	20.5 ± 1.5		3.7 ± 0.1	0.028		0.9 ± 0.1
Cat	52			0.068		
Water		72	63 ± 2	0.0038	0.91	5 – 6

than the difference between individual samples of one kind. A few observations were made on liver at λ 3.2 cm. The results lay within the range for muscle. The experimental uncertainty in the determination of the absorption coefficient of muscle at λ 3.2 cm. was very large. The data suggested that the absorption was slightly greater than that for water, but this is not certain. The fat used was free from noticeable amounts of blood. The effect of the presence of this was marked, as shown by the results with one sample of rabbit fat that was tested, with the note "much connective tissue, looks very bloody". The ϵ was 43, and the absorption coefficient 0.05, both much larger than the values given in the table. Similar effects on the values obtained for bone marrow were noted.

Some observations were made on the effect of temperature on the values of the constants for muscle and fat at λ 1.72 m. The dielectric constant increased about one unit per 1° C. for both frog and beef muscle. The data on the absorption coefficient are less clear, but indicate an increase of 0.005 per 10° C. For fat no change was observed greater than the experimental error for a rise of temperature of 10° C.

From a comparison of the results at different wave lengths, it is evident that there is a larger proportional increase in the absorption by muscle than by fat in going from λ 1.72 m. to the microwave region. This means a large increase in the relative heating of muscle in a radiation field, but at the same time a reduction in penetration. Although there is large uncertainty in the values given for the absorption coefficient, it is safe to say that at a depth of $2\frac{1}{2}$ mm. the intensity of radiation at wave length 3.2 cm. will be reduced to one-quarter of its value at the surface, or that three-quarters of the energy entering such a material will be absorbed in this layer. The reduction in the dielectric constant is more marked for muscle, and this is important, as the amount of reflection from a surface is thereby reduced. It may be noted

that values for bone marrow are very similar to those for fat, except for the dielectric constant at λ 1.72 cm. The dielectric constant of bone appears to change little over the range covered, but the absorption is much greater at λ 3.2 cm.

Table II shows some results reported by others on the dielectric constant and absorption coefficient of comparable material. The values of 2α attri-

TABLE II

VALUES OF ϵ AND 2α BY DIFFERENT OBSERVERS

		Muscle		Fat		Bone marrow		Bone	
	λ	ϵ	2α	ϵ	2α	ϵ	2α	ϵ	2α
Schereschewsky (13)	1.2 – 2.8 m.	77.6		13.6					
Osswald (11)	3 m.	71	0.26	12	0.039	7.3	0.028		
Authors	1.7 m.	148	0.09	9.5	0.026	20	0.028		
		135	0.08	23	9.036	52	0.068		
Dakin and Works (7)	10 cm.	62.6	1.48				λ3	6.98	0.516
Authors	9.5 cm.	54	1.54				cm.	6.86	0.562

buted to Osswald have been computed from the values given by him for the conductivity σ from the relation $2\alpha = \frac{120\pi\sigma}{n}$. The present values are repeated for comparison. The differences are not greater than could be attributed to differences in the material used except in the case of muscle at λ 1.72 cm. Partly because of this difference, the theory of the method used was re-examined, and corrections were applied to the equations used by earlier investigators as indicated. Sources of error other than those discussed earlier, and in the appendix, have not suggested themselves. The error due to the effect of the current in the thermocouple bridge when the circuit is not in resonance has not lent itself to mathematical correction, and another method for the observation of resonance would be preferable.

There were some cases observed at λ 1.72 m. where the properties of muscle seemed to change more than could be attributed to heating, and later a long study was made on a variety of samples. The result showed that the earlier observations were due to a slight change in the experimental conditions, and no change in properties was found that could not be accounted for by the heating of the material.

The measurements on water were not made with special care, but for comparison sake, and are included because of the general importance of this medium. A review of data on water was given by Knerr (8) in 1937. Some of the values that have appeared since then are given in Table III, together

TABLE III

VALUES FOR WATER BY DIFFERENT OBSERVERS

	λ	ϵ	2α		λ	ϵ	2α
Slevogt (15)	246 cm.	81	0.00167	Abadie (2)	11 cm.	78	0.7
	58.3	81	0.030		3	65	8.2
	10.4	78.4	0.81				
Bäz (3)	9.35	77.5	0.91	Collie, Ritson and			
	2.8	66.7	9.8	Hasted (4)	10	77.4	0.93
Authors	172		0.0038				
	9.5	72	0.91	Roberts and von			
	3.2	63	5 – 6	Hippel (12)	6	77	1.9

with the values obtained with the equipment here. The values ascribed to Abadie were taken from a graph, and, since no two of those quoted have used the same form in stating results, these have been reduced as shown.

In general the data reported should give a fairly good estimate of the relative amount of energy that would be absorbed by different layers of such biological material if one knows the thickness and general composition. It does not appear to the authors that it would be of much value to try to improve on the measurements, because of such big differences in tissues of one type, and because in the microwave region absorption by individual ingredients is so masked by the strong absorption of water; on the other hand, similar measurements of the properties of the separate ingredients might produce interesting results.

Note

After this report was written, a note by T. S. England and N. A. Sharples on "Dielectric Properties of the Human Body in the Microwave Region of the Spectrum" appeared in *Nature* of March 26th. The results are given in the form of data on α and β as obtained from the propagation factor in the wave guide for a wave length 3.18 cm. Table IV shows their results and the present ones at λ 3.2 cm. for comparable material.

TABLE IV

Material	England and Sharples		Authors	
	α	β	α	β
Muscle	2.5 – 2.7	11.6 – 11.7	2.5 – 3.3	10.8 – 12.5
Fat	0.44 – 0.49	4.0	0.37 – 0.42	3.5 – 4.2
Bone	0.56	5.4	0.29 – 0.30	5.0
Bone marrow	0.79	5.1	0.33 – 0.51	3.5 – 3.7

On the whole the results suggest that animal tissue, even when from refrigeration, not frozen, has very much the same properties as the comparable human tissue. It was realized that the samples of bone used here had air-dried for several days, and this measurement has been repeated here with a fresh sample, giving $\alpha = 0.48$ $\beta = 5.4$. It was possible to add a measurement on samples of baby mice skin, and values of $\alpha = 2.4$ and 2.5 and of $\beta = 11.1$ and 13 were found, of the same order as those reported in *Nature*.

It was noted that this skin was less than 1 mm. thick, and that there are parts of the human body where the skin is thin, and to which the comment on the absorption in skin would not apply. It has been suggested elsewhere that a technique of irradiating a tissue for short intervals would avoid the undue heating of general tissue, if it should appear that any particular frequency has a desirable selective effect on some special ingredient.

Acknowledgments

We are greatly indebted to Dr. Helen Battle of the Department of Zoology for providing the prepared fresh material used, and for her unfailing kindness in giving information on biological questions. The work at λ 9.5 cm. and most of that at λ 1.72 m. was done during 1946–47 by the junior author on a grant from the Ontario Cancer Research Foundation, which has also supplied funds for some technical assistance and equipment. We wish to thank Dr. R. C. Dearle for the use of space and the facilities of the Department of Physics and for his interest in the progress of the research at a time when there were so many demands on both.

References

1. Abadie, P. Onde Elect. 69 : 247. 1937.
2. Abadie, P. Trans. Faraday Soc. 42A : 143. 1946.
3. Bäz, G. Physik. Z. 40 : 394. 1939.
4. Collie, C. H., Ritson, D. M., and Hasted, J. B. Trans. Faraday Soc. 42A : 129. 1946.
5. Cooledge, W. D. Ann. Physik, 69 : 125. 1899.
6. Cripwell, F. J. and Sutherland, G. B. B. M. Trans. Faraday Soc. 42A : 149. 1946.
7. Dakin, T. W. and Works, C. N. J. Applied Phys. 18 : 789. 1947.
8. Knerr, H. W. Phys. Rev. 52 : 1054. 1937.
9. McLennan, J. C. and Burton, A. C. Can. J. Research, 3 : 224. 1930.
10. McLennan, J. C. and Burton, A. C. Can. J. Research, 5 : 550. 1931.
11. Osswald, K. Hochfrequenztech. Elektroakust. 49 : 40. 1937.
12. Roberts, S. and von Hippel, A. J. Applied Phys. 17 : 610. 1946.
13. Schereschewsky, J. W. U.S. Pub. Health Service. Pub. Health Repts. 48 : 844. 1933.
14. Slätis, H. Ann. Physik, 36 : 397. 1939.
15. Slevogt, K. Ann. Physik, 36 : 141. 1939.

(For Appendix see next page).

APPENDIX

The circuit as used at 1.7 m. was taken as equivalent to that in Fig. 4, where B' is a sliding bridge of zero impedance, and B a fixed bridge. At A

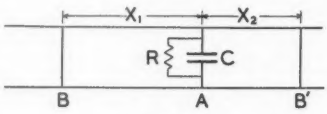

FIG. 4.

the impedance is composed of R, $\frac{1}{j\omega C}$, and $Z \tanh \gamma x_2$ in parallel, where Z is the characteristic line impedance, and $\gamma = \alpha + j\beta$ is the propagation constant. Using the value obtained thus as the terminal impedance of the section BA, the input impedance at B is found,

$$Z_B = Z \frac{1 + \tanh \gamma x_1 \left(\frac{Z}{R} + \coth \gamma x_2 + j\omega C Z_1\right)}{\tanh \gamma x_1 + \frac{Z}{R} + \coth \gamma x_2 + j\omega C Z_1}.$$

If the effective P.D. across the line at B is constant, the mean square current flowing in the line at B, and hence the thermocouple response in B, will be proportional to

$$\left|\frac{Z}{Z_B}\right|^2 = U.$$

Neglecting terms beyond the first order in α ($\alpha \gg 4.1 \times 10^{-5}\text{cm.}^{-1}$) one finds

$$(6)\quad U = \frac{(1 + \omega C Z_1 \cot\theta_1 - \cot\theta_1 \cot\theta_2)^2 + \frac{Z^2}{R_2}\cot^2\theta_1 + \frac{2\alpha Z}{R}\left(\frac{x_1}{\sin^2\theta_1} + \frac{x_2 \cot^2\theta_1}{\sin^2\theta_2}\right)}{(\cot\theta_1 + \cot\theta_2 - \omega C Z)^2 + \left(\frac{Z}{R} + \frac{\alpha x_1}{\sin^2\theta_1} + \frac{\alpha x_2}{\sin^2\theta_2}\right)^2},$$

where $\theta_1 = \beta x_1$, $\theta_2 = \beta x_2$. In this expression for U, x_2 or θ_2 is the only variable and the condition for a maximum may be found from $\frac{\delta U}{\delta x_2} = 0$. Writing $\cot\theta_1 + \cot\theta_2 - \omega C Z = A$, this yields

$$\cot\theta_1 \left(A^2 - A\frac{1 + \cot^2\theta_1}{\cot\theta_1} - \frac{Z^2}{R^2}\right) + A\frac{2\alpha Z}{R}\left[x_1\left(\cot^2\theta_1 - 1\right) + \cot\theta_1\left(2x_2 \cot\theta_2 - \frac{1}{\beta}\right)\right]$$
$$- \frac{2\alpha Z}{R}\cot\theta_1\left(\frac{x_1}{\sin^2\theta_1} + \frac{x_2}{\sin^2\theta_2}\right) - \frac{\alpha Z}{R}\left(1 + \cot^2\theta_1\right)\left(2x_2 \cot\theta_2 - \frac{1}{\beta}\right) = 0.$$

In all cases met with, the coefficient of A in the second term is much less than that in the first term, and may be neglected. Dividing through by $\cot\theta_1$ one may then write

$$(7)\quad A^2 - A\frac{1 + \cot^2\theta_1}{\cot\theta_1} - \triangle = 0,$$

where

$$\triangle = \frac{Z^2}{R^2} + \frac{2\alpha Z}{R}\left(\frac{x_1}{\sin^2\theta_1} + \frac{x_2}{\sin^2\theta_2}\right) + \frac{\alpha Z}{R}$$
$$\left(\cot\theta_1 + \tan\theta_1\right)\left(2x_2\cot\theta_2 - \frac{1}{\beta}\right).$$

Since $\frac{1+\cot^2\theta_1}{\cot\theta_1} \not< 2$, and becomes infinite for $x_1 = \frac{\lambda}{4}$, while $\triangle$ is always small, <0.02 in the present experiments except for muscle, where it may be as much as 0.2, the approximate solution of Equation (7) may be used,

$A = -\triangle\frac{\cot\theta_1}{1+\cot^2\theta_1}$, and except for materials of relatively high conductivity A may be put equal to zero. The equation

$$\omega CZ = \cot\theta_1 + \cot\theta_2 - A \tag{8}$$

yields the capacity C, and hence the dielectric constant.

To obtain an expression for the absorption index κ, returning to Equation (6) one has

$$U_{\max} = \frac{[1+\cot\theta_1(\cot\theta_1 - A)]^2 + \frac{Z^2}{R^2}\cot^2\theta_1 + \frac{2\alpha Z}{R}\left(\frac{x_1}{\sin^2\theta_1} + \frac{x_2\cot^2\theta_1}{\sin^2\theta_2}\right)}{\left(\frac{Z}{R} + \frac{\alpha x_1}{\sin^2\theta_1} + \frac{\alpha x_2}{\sin^2\theta_2}\right)^2 + A^2},$$

which for the case of no container present, $R = \infty$, $A = 0$ becomes

$$\left(\frac{2}{\alpha\lambda}\right)^2 = \left(\frac{2}{\delta}\right)^2.$$

In the numerator the last term is always negligible in comparison with the first, and in both numerator and denominator the second term is small in comparison with the first. Hence, if the maximum galvanometer deflections are called G_1 and G_2

$$\sqrt{\frac{G_1}{G_2}} = \frac{2}{\delta}\cdot\frac{\frac{Z}{R} + \frac{\alpha x_1}{\sin^2\theta_1} + \frac{\alpha x_2}{\sin^2\theta_2} + \frac{1}{2}A^2\Big/\left(\frac{Z}{R} + \frac{\alpha x_1}{\sin^2\theta_1} + \frac{\alpha x_2}{\sin^2\theta_2}\right)}{1 + \cot\theta_1(\cot\theta_1 - A) + \frac{1}{2}\frac{Z^2}{R^2}\cot^2\theta_1\Big/[1+\cot\theta_1(\cot\theta_1 - A)]}$$

$$= \frac{2}{\delta}\frac{\frac{Z}{R} + \frac{\alpha x_1}{\sin^2\theta_1} + \frac{\alpha x_2}{\sin^2\theta_2} + D_1}{1 + \cot^2\theta_1 + D_2}, \tag{9}$$

where D_1 and D_2 are small corrections.

From general electromagnetic theory $\sigma = \frac{n^2\kappa}{30\lambda}$, and $\epsilon = n^2(1-\kappa^2)$, where κ is the index of absorption defined by Drude; also $\frac{1}{R} = 4\pi C_0\sigma$, where C_0 is the internal capacity of the empty container in e.s.u., it being assumed that the electric field in the container has the same form in all cases.

Hence

$$\frac{Z}{R} = \frac{\cot\theta_1 + \cot\theta_2 - A}{\omega C_{\text{farad}}} \frac{4\pi n^2 \kappa C_0}{30\lambda}$$

$$= \frac{\cot\theta_1 + \cot\theta_2 - A}{C' + \epsilon C_0} \cdot \frac{2\kappa\epsilon C_0}{1-\kappa^2},$$

where $C' + \epsilon C_0$ is the total capacity of the container filled with the medium of dielectric constant ϵ.

Inserting this value of $\frac{Z}{R}$ in Equation (9) and rearranging, one finds

$$\frac{4\kappa}{1-\kappa^2} = \frac{C' + \epsilon C_0}{\epsilon C_0} \frac{\delta}{\cot\theta_1 + \cot\theta_2 - A} \left[(1 + \cot^2\theta_1 + D_2)\sqrt{\frac{G_1}{G_2}} - \frac{2}{\lambda}\left(\frac{x_1}{\sin^2\theta_1} + \frac{x_2}{\sin^2\theta_2}\right) - \frac{2D_1}{\delta}\right]. \tag{10}$$

This gives κ, and, except where $n^2\kappa$ is large, A, D_1, and D_2 may be omitted as in Equation (2).

A NEW HIGH LIGHT GATHERING POWER SPECTROGRAPH FOR AURORAL STUDIES[1]

By W. Petrie

Abstract

The optics of the spectrograph consist of two *f* 2.5, 12 in. aircraft camera lenses, and a 15,000 lines per inch plane grating with ruled surface 4 in. by 4 in. The grating concentrates light in the second order spectrum; the dispersion in this order for normal incidence is 22 Å per mm. A few spectra were obtained during the spring of 1949, and the results appear to be most promising. These spectra include several interesting features, one of which is a strong radiation of wave length 4320 Å in the second order. This radiation is likely to be an intense first order infrared feature.

Introduction

There is still great need for moderate dispersion spectrograms of the aurora if we are to make full use of this phenomenon in interpreting physical conditions in the upper atmosphere. Little is known regarding the near infrared region of the spectrum, and it is especially important that we determine the intensities of numerous faint features that have been attributed to atomic oxygen and nitrogen in several stages of ionization. It is hoped that information on these and other problems will be obtained with a new spectrograph constructed at the University of Saskatchewan.

Description of the Instrument

Two aircraft camera units were purchased from U.S. War Assets. The camera lenses, film holders, and camera back are all used in the spectrograph. The lenses are 7-element, *f* 2.5, 12 in. focal length Ektar lenses manufactured by the Eastman Kodak Co. One lens is used as collimator and covers a section of the sky that subtends an angle of approximately 22° at the camera. All inner air–glass surfaces in these lenses are coated with a nonreflecting film that minimizes the loss of light by reflection and prevents the formation of "ghost" images of appreciable intensity. According to the literature (1), the focal plane shift for a lens of this type is approximately 0.01 mm. per 1° C. change in temperature. The few observations made to date suggest that thermostatic control of the instrument will not be necessary.

The dispersing element is a plane grating obtained from Johns Hopkins University. The ruled surface is 4 in. by 4 in., there are 15,000 lines per in., and the light is concentrated in the second order spectrum. The grating and camera lens produce a second order dispersion for normal incidence of 22 Å per mm.

The spectrograph components are mounted on an aluminum base plate and surrounded with a three ply plywood housing. The lid may be removed,

[1] *Manuscript received August 9, 1949.*

Contribution from the Department of Physics, University of Saskatchewan, Saskatoon, Sask.

and a 5 in. by 10 in. opening is provided on one side for loading and removing the plate holder. A 5 in. diameter circular section is cut out of an adjacent side of the housing, so that the slit and mounting may be exposed to the night sky. Both these openings can be covered from the outside by tin slides. The interior of the housing is painted black, and suitable baffles are provided to shield the optical components and the photographic plate from the regions of the first and third order spectra which are not incident on the camera lens.

A shutter mounted behind the slit may be opened and closed at specified times by the use of an electric time clock. This device makes possible the operation of the spectrograph during an auroral display without the presence of an operator.

In order that the instrument may readily be pointed at different regions of the sky, the spectrograph box is clamped to a stand in such a way that the altitude of the instrument may be varied from 0° to 90°. The stand is fitted with castors and sits on an 8 ft. by 8 ft. wooden platform on the roof of the Physics Building. This arrangement enables the stand to be turned to any direction. Figs. 1 and 2 illustrate the interior and mounting of the spectrograph.

Auroral Spectrograms Obtained with the Instrument

The spectrograph was used for the first time during the night of February 21, 1949. On this date, an intense auroral display was observed at Saskatoon (2). A four hour exposure yielded a spectrogram that showed 18 measurable features, and a number of lines or bands too weak for wave length measurement. Six other spectrograms were obtained between February 24 and April 1, but none of these were as dense as the first spectrogram secured on February 21. A list of the more intense lines and bands present in the strongest spectrogram is given in Table I.

TABLE I

Wave length, I.A.	Identification	Wave length, I.A.	Identification
3805	N_2, 2 P.G.	5577	OI, $^1D_2 - {}^1S_0$
3915	N_2, N.G.	5675	?
4278	N_2, N.G.	5867	N_2, 1 P.G.
4320	?	5993	N_2, 1 P.G.
4652	N_2, N.G.	6300	OI, $^3P_2 - {}^1D_2$
4709	N_2, N.G.	6364	OI, $^3P_1 - {}^1D_2$
4861	H_β	6563	H_α
4992	?	6676	?
5202–5278	?	6753	?

The most interesting feature of these spectra is the appearance of a relatively strong radiation at a wave length of 4320 Å. This radiation is about as intense as the nitrogen band at λ 4709, and does not correspond to any line or band given in Vegard's tables (3) of auroral wave lengths. The writer is of the

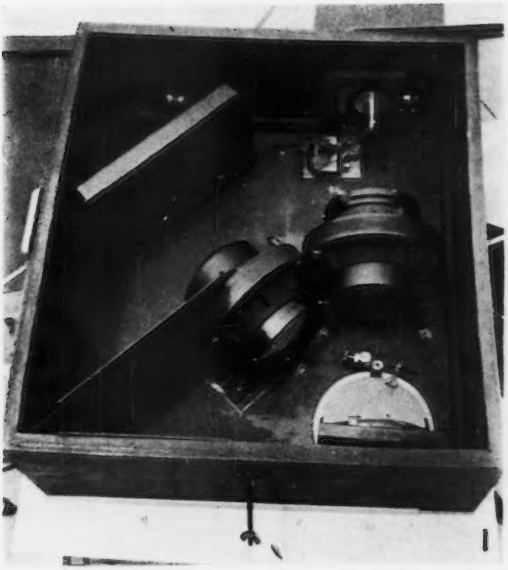

FIG. 1. *The interior of the spectrograph.*

FIG. 2. *The mounting of the spectrograph.*

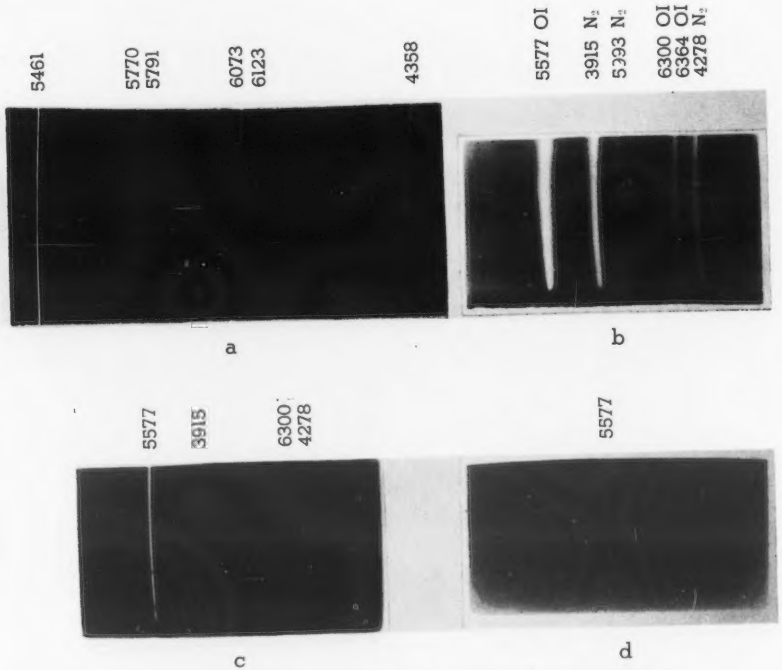

FIG. 3. *(a) The mercury arc spectrum photographed with the new instrument.*
(b) Aurora, February 21, 1949. Exposure, four hours; slit, 1 mm.
(c) Aurora, March 1, 1949. Exposure, four hours; slit, 0.25 mm.
(d) Aurora, March 2, 1949. Exposure, 15 min.; slit, 0.25 mm.

opinion that this feature of unknown origin belongs to the first order infrared spectrum. It is our intention to obtain second order infrared auroral spectra about which little is known.

It is well to note that the first two members of the hydrogen Balmer series appear in the February 21 spectrogram. Also present are several features about whose origin the writer is undecided. A stronger spectrogram is needed before the identification of these features can be reasonably certain. Fig. 3 is a reproduction of parts of several auroral spectrograms taken with the new instrument.

The results obtained to date appear to be promising, and the instrument will be used chiefly to secure infrared spectra of the aurora. This region of the spectrum has not been examined in any detail and much remains to be done. All future plates will be calibrated with a step sector disk so that line intensities may be obtained.

Acknowledgment

The construction and operation of this instrument has been supported by a grant from the National Research Council of Canada.

References

1. KINGSLAKE, R. J. Optical Soc. Am. 37 : 1. 1947.
2. PETRIE, W., FORSYTH, P. A., and MCCONECHY, E. Nature, 163 : 774. 1949.
3. VEGARD, L. and KVIFTE, G. Geofys. Publikasjoner Norske Videnskaps-Akad. Oslo, 16 (7) : 13. 1945.

CANADIAN JOURNAL OF RESEARCH

Notes on the Preparation of Copy

GENERAL:—Manuscripts should be typewritten, double spaced, and the **original and one extra copy** submitted. Style, arrangement, spelling, and abbreviations should conform to the usage of this Journal. Names of all simple compounds, rather than their formulae, should be used in the text. Greek letters or unusual signs should be written plainly or explained by marginal notes. Superscripts and subscripts must be legible and carefully placed. Manuscripts should be carefully checked before being submitted, to reduce the need for changes after the type has been set. If authors require changes to be made after the type is set, they will be charged for changes that are considered to be excessive. **All pages, whether text, figures, or tables, should be numbered.**

ABSTRACT:—**An abstract of not more than about 200 words, indicating the scope of the work and the principal findings, is required.**

ILLUSTRATIONS:

(i) *Line Drawings:*—All lines should be of sufficient thickness to reproduce well. Drawings should be carefully made with India ink on white drawing paper, blue tracing linen, or co-ordinate paper **ruled in blue only;** any co-ordinate lines that are to appear in the reproduction should be ruled in black ink. Paper ruled in **green, yellow, or red should not be used** unless it is desired to have all the co-ordinate lines show. Lettering and numerals should be neatly done in India ink preferably with a stencil **(do not use typewriting)** and be of such size that they will be legible and not less than one millimeter in height when reproduced in a cut three inches wide. All experimental points should be carefully drawn with instruments. Illustrations need not be more than two or three times the size of the desired reproduction, but the ratio of height to width should conform with that of the type page. **The original drawings and one set of small but clear photographic copies are to be submitted.**

(ii) *Photographs:*—Prints should be made on glossy paper, with strong contrasts; they should be trimmed to remove all extraneous material so that essential features only are shown. Photographs should be submitted **in duplicate;** if they are to be reproduced in groups, one set should be so arranged and mounted on cardboard with rubber cement; the duplicate set should be unmounted.

(iii) *General:*—**The author's name, title of paper, and figure number should be written in the lower left-hand corner (outside the illustration proper) of the sheets on which the illustrations appear.** Captions should not be written on the illustrations, but typed on a separate page of the manuscript. All figures (including each figure of the plates) should be numbered consecutively from 1 up (arabic numerals). **Each figure should be referred to in the text.** If authors desire to alter a cut, they will be charged for the new cut.

TABLES:—Titles should be given for all tables, which should be numbered in Roman numerals. Column heads should be brief and textual matter in tables confined to a minimum. **Each table should be referred to in the text.**

REFERENCES:—**These should be listed alphabetically by authors' names, numbered in that order, and placed at the end of the paper.** The form of literature citation should be that used in the respective sections of this Journal. **Titles of papers should not be given in references listed in Sections A, B, E, and F, but must be given in references listed in Sections C and D.** The first page only of the references cited in papers appearing in Sections A, B, and E should be given. **All citations should be checked with the original articles.** Each citation should be referred to in the text by means of the key number; in Sections C and D the author's name and the date of publication may be included with the key number if desired.

The *Canadian Journal of Research* conforms in general with the practice outlined in the *Canadian Government Editorial Style Manual*, published by the Department of Public Printing and Stationery, Ottawa.

Reprints

Fifty reprints of each paper without covers are supplied free. Additional reprints, if required, will be supplied according to a prescribed schedule of charges. On request, covers can be furnished at cost.